

Nonlinearity enhancement in optomechanical system

Ling Zhou,¹ Jiong Cheng,¹ Yan Han,¹ and Weiping Zhang²

¹*School of physics and optoelectronic technology, Dalian University of Technology, Dalian 116024, P.R.China*

²*State Key Laboratory of Precision Spectroscopy, Department of Physics,
East China Normal University, Shanghai 200062, P.R. China*

The nonlinearity is an important feature in the field of optomechanics. Employing atomic coherence, we put forward a scheme to enhance the nonlinearity of the cavity optomechanical system. The effective Hamiltonian is derived, which shows that the nonlinear strength can be enhanced by increasing the number of atoms at certain range of parameters. We also numerically study the nonlinearity enhancement beyond the effective Hamiltonian. Furthermore, we investigate the potential usage of the nonlinearity in performing quantum nondemolition (QND) measurement of the bosonic modes. Our results show that the present system exhibits synchronization, and the nonlinear effects provide us means in performing QND.

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I. INTRODUCTION

Optomechanical system by coupling mechanical resonators to the fields of optical cavities can provide us available device to observe quantum mechanical behaviors of macroscopic system. It has been proved that entanglement of two resonators and of cavity and mirror can be generated by radiation pressure [1–4], while the pressure can be used to cool down the mirrors [5–9], and the analogy electromagnetically induced transparency phenomenon may happen in cavity optomechanical system [10–13] and has demonstrated in experiment [14]. In addition, optomechanical system can be used as transducers for long-distance quantum communication [15, 16].

On the other hand, the observation of strict quantum effects in quantum optics relies on the existence of strong nonlinear interaction between photons [17]. Unfortunately, photons tend to interact only weakly, thus enhancement of photon-photon interaction at the few-photon level is still a challenge in quantum optics. A lot of efforts are devoted to enhance the nonlinearity of photons [18–21]. Gong et. al. [22] has shown us that an optomechanical system can lead to nonlinear Kerr effect, but it is very weak (proportion to $\frac{G^2}{\omega_m^2}$) because usually the radiation pressure coupling strength G is less than the

frequency of the oscillator ω_m for weak coupling system. Most recently, Ludwig et. al. [23] propose a scheme to enhance cross-Kerr nonlinearity in double cavities with membrane in the middle where the tunnel rate between cavities weaken the negative influence of large value of the oscillator frequency.

Optomechanics experiments are rapidly approaching the single-photon strong-coupling regime $G \geq \omega_m$ [25]. However, for weakly driven systems [24, 25], Rabl [24] has shown that photon blockade under single-photon strong coupling condition is affected by $\frac{G}{\omega_m}$ where for $\frac{G}{\omega_m} > \frac{1}{2}$, no significant further improvement of the nonlinearity is achieved, and the nonlinear effects are suppressed by $\frac{G}{\omega_m}$. Thus, improving the nonlinearity beyond strong-coupling means deserves our investigation. In this paper, we consider weakly driven system with weak coupling $G < \omega_m$ and $G^2 < \kappa\omega_m$ and put forward an alternative scheme to enhance and to modulate the photon-photon and photon-phonon cross-Kerr nonlinearity by employing atomic coherence. We also show the nondemolition measurement of phonon and photon. Comparing the scheme with [23–25], the photon-photon and photon-phonon cross-Kerr nonlinearities not only can be enhanced but also can be controlled. We do not need single-photon strong coupling condition, which means that our scheme maybe easier to be realized.

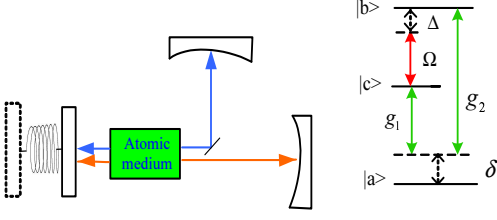


FIG. 1: (Color online) Sketch of the system and the atomic configuration. Two-mode cavity fields interact with atomic transitions $|a\rangle \leftrightarrow |c\rangle$ and $|a\rangle \leftrightarrow |b\rangle$ with detuning δ , while the classical field drive the atomic level between $|b\rangle \leftrightarrow |c\rangle$ with detuning Δ .

II. MODEL AND EFFECTIVE INTERACTION

We consider atomic media trapped in a doubly resonant cavity with one partially transmitting fixed mirror and one movable mirror (see Fig. 1). The two cavity modes with frequencies ω_1 and ω_2 couple to atomic transition $|a\rangle \leftrightarrow |c\rangle$ and $|a\rangle \leftrightarrow |b\rangle$ with the same detuning δ , and the classical laser field with Rabi frequency Ω interacts with the atoms between the transition $|b\rangle \leftrightarrow |c\rangle$ with detuning Δ . Our model is similar with quantum beat laser [26] except with one movable mirror. The Hamiltonian of the hybrid system is given by

$$H = H_{af} + H_{fo} + H_{dr}.$$

The free energy of fields and the atoms as well as their interaction is

$$H_{af} = \sum_{j=1,2} \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_{k=1}^N \left[\sum_{i=a,b,c} E_i \hat{\sigma}_{ii}^{(k)} + (g_1 \hat{a}_1 \hat{\sigma}_{ca}^{(k)} + g_2 \hat{a}_2 \hat{\sigma}_{ba}^{(k)} + \Omega e^{-i\omega_\Omega t} \hat{\sigma}_{cb}^{(k)} + h.c.) \right], \quad (1)$$

The first term describes the energy of the two cavity modes with lowering operator a_j and cavity frequency ω_j (at equilibrium position). The second term represents

the energy of the atoms and the interaction between the atoms and the cavity fields, where $\sigma_{ij} = |i\rangle\langle j|$ is the spin operator of the atoms, g_j is the coupling between the cavity and the atoms, and Ω is the Rabi frequency of the classical field driving the atoms between $|c\rangle$ and $|b\rangle$.

$$H_{fo} = \omega_m \hat{b}^\dagger \hat{b} + G(\hat{a}_1 + \hat{a}_2)^\dagger (\hat{a}_1 + \hat{a}_2) (\hat{b} + \hat{b}^\dagger), \quad (2)$$

where the first term is the free energy of the mechanical oscillator with frequency ω_m , and the second term represents the coupling between the non-polarized two-mode fields and the mechanical resonator with radiation-pressure coupling G , and the form of the coupling has been employed to cool the mirror in [6]. We will show that the coupling among the cavity fields and the resonator can enhance the nonlinearity.

$$H_{dr} = \sum_{j=1,2} \varepsilon_j (\hat{a}_j^\dagger e^{-i\omega_{L_j} t} + \hat{a}_j e^{i\omega_{L_j} t}) \quad (3)$$

describes the two-mode cavity fields driven by weak classical fields.

Now, we switch into interaction picture rotating with $H_0 = \sum_{j=1,2} \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_{k=1}^N [\sum_{i=a,b,c} E_i \hat{\sigma}_{ii}^{(k)} + \delta \hat{\sigma}_{aa}^{(k)} - \Delta \hat{\sigma}_{cc}^{(k)}]$ where $\Delta = E_b - E_c - \omega_\Omega$, $\delta = E_c - E_a - \omega_1 = E_b - E_a - \omega_2$. Then

$$H_{af1} = \sum_k [\Delta \hat{\sigma}_{cc}^{(k)} - \delta \hat{\sigma}_{aa}^{(k)} + (g_1 \hat{a}_1 \hat{\sigma}_{ca}^{(k)} + g_2 \hat{a}_2 \hat{\sigma}_{ba}^{(k)} + \Omega \hat{\sigma}_{cb}^{(k)} + h.c.)], \quad (4)$$

$$H_{fo1} = \omega_m \hat{b}^\dagger \hat{b} + G(\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2 e^{-idt} + \hat{a}_2^\dagger \hat{a}_1 e^{idt})(\hat{b} + \hat{b}^\dagger), \quad (5)$$

$$H_{dr1} = \sum_{j=1,2} \varepsilon_j (\hat{a}_j^\dagger + \hat{a}_j), \quad (6)$$

where $d = \omega_2 - \omega_1$ is the frequency difference between the two cavity modes. For simplicity, we have assume

$\omega_j = \omega_{L_j}$ ($j = 1, 2$). One can see that the Hamiltonian (5) contains the terms $\hat{a}_1^\dagger \hat{a}_2 \hat{b} + h.c.$ and $\hat{a}_2^\dagger \hat{a}_1 \hat{b} + h.c.$ which are the typically nondegenerate parametric amplification. That means, under certain range of parameters, one can obtain squeezed states between one of the cavity modes and the mechanical resonator. In addition, it had been shown in [27] that the output field exhibits squeezing for single mode cavity optomechanics system under strong drive condition. For the present coupling system, the squeezing properties of the cavity fields deserve our further investigation using linearized theory under strongly driven condition. But, we here focus on the nonlinearity enhancement and show that the form of the coupling in (5) plays an important role for enhancement of the nonlinearity.

Firstly, we consider large detuning condition and derive the effective Hamiltonian of H_{af1} (4), and the motions for the atomic operators $\hat{\sigma}_{ac}^k$ and $\hat{\sigma}_{ab}^k$ are given by

$$\begin{aligned} i \frac{d\hat{\sigma}_{ac}^k}{dt} &= (\delta + \Delta) \hat{\sigma}_{ac}^k + g_1 \hat{a}_1 (\hat{\sigma}_{aa}^k - \hat{\sigma}_{cc}^k) - g_2 \hat{a}_2 \hat{\sigma}_{bc}^k + \Omega \hat{\sigma}_{ab}^k, \\ i \frac{d\hat{\sigma}_{ab}^k}{dt} &= \delta \hat{\sigma}_{ab}^k - g_1 \hat{a}_1 \hat{\sigma}_{cb}^k + g_2 \hat{a}_2 (\hat{\sigma}_{aa}^k - \hat{\sigma}_{bb}^k) + \Omega \hat{\sigma}_{ac}^k. \end{aligned} \quad (7)$$

Under the large detuning conditions $\delta \gg \{g_1, g_2\}$, $\Delta \gg \Omega$, Eq. (7) can be solved adiabatically by taking $d\hat{\sigma}_{ac}^k/dt = d\hat{\sigma}_{ab}^k/dt = 0$. The adiabatic solutions of $\hat{\sigma}_{ac}^k$ and $\hat{\sigma}_{bc}^k$ can then be substituted into the Hamiltonian (4). The most of the atoms are in their ground state $|a\rangle$, thus, by elimination of the atomic variables the effective Hamiltonian describing the interaction between the two mode fields can be written as

$$\begin{aligned} H_{af2} &= -\nu_1 \hat{a}_1^\dagger \hat{a}_1 - \nu_2 \hat{a}_2^\dagger \hat{a}_2 \\ &\quad + \lambda (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1), \end{aligned} \quad (8)$$

where

$$\begin{aligned} \nu_1 &= \frac{2g_1^2 \delta N}{\tilde{\Delta}}, \nu_2 = \frac{2g_2^2 (\delta + \Delta) N}{\tilde{\Delta}}, \\ \lambda &= \frac{2g_1 g_2 \Omega N}{\tilde{\Delta}}, \end{aligned}$$

with $\tilde{\Delta} = \delta(\Delta + \delta) - \Omega^2$. We see that an additional interaction term between the two-mode cavity fields is introduced because of the interaction between atomic media and the fields. As shown in Fig.1, the two-mode fields interact with the atoms between the transitions $|a\rangle \leftrightarrow |b\rangle$ and $|a\rangle \leftrightarrow |c\rangle$ (similar with V-type configuration). Even though most of the atoms are in their ground state, the participation of the atoms still induce a beam splitter type interaction shown in Eq.(8). It is the term proportional to the number of atoms that enhances the cross-Kerr nonlinearity.

Although we study the large detuning case, the value of ν_i and λ can be large, because it is proportional to the number of atoms; therefore we can switch into a picture rotating with $\omega_m \hat{b}^\dagger \hat{b} + H_{af2}$ and treat the other terms as perturbation. In order to do that, we diagonalize the Hamiltonian (8) by defining $\hat{c}_1 = \hat{a}_1 \cos \theta + \hat{a}_2 \sin \theta$ and $\hat{c}_2 = \hat{a}_1 \sin \theta - \hat{a}_2 \cos \theta$, and then we have

$$H_{af3} = -\omega_{c1} \hat{c}_1^\dagger \hat{c}_1 - \omega_{c2} \hat{c}_2^\dagger \hat{c}_2, \quad (9)$$

with

$$\begin{aligned} \omega_{c1} &= \nu_1 \cos^2 \theta + \nu_2 \sin^2 \theta - \lambda \sin 2\theta, \\ \omega_{c2} &= \nu_1 \sin^2 \theta + \nu_2 \cos^2 \theta + \lambda \sin 2\theta, \\ tg2\theta &= \frac{2\lambda}{\nu_2 - \nu_1}. \end{aligned} \quad (10)$$

Now we jointly consider Hamiltonian (5) and (9) and switch into a picture rotating with $H'_0 = \omega_m \hat{b}^\dagger \hat{b} - \omega_{c1} \hat{c}_1^\dagger \hat{c}_1 - \omega_{c2} \hat{c}_2^\dagger \hat{c}_2$. Then, we use the effective Hamiltonian method proposed in [28] and have the effective Hamiltonian as

$$\begin{aligned} H_{eff} &= \eta_1 \hat{c}_1^\dagger \hat{c}_1 \hat{c}_2^\dagger \hat{c}_2 + \eta_2 (\hat{c}_1^\dagger \hat{c}_1 - \hat{c}_2^\dagger \hat{c}_2) \hat{b}^\dagger \hat{b} + \\ &\quad + s[(\hat{c}_1^\dagger \hat{c}_1)^2 + (\hat{c}_2^\dagger \hat{c}_2)^2] + u_1 \hat{c}_1^\dagger \hat{c}_1 + u_2 \hat{c}_2^\dagger \hat{c}_2, \end{aligned} \quad (11)$$

where

$$\begin{aligned}
\eta_1 &= v + u_2 - u_1, \eta_2 = u_2 - u_1, \\
v &= \left[\frac{\omega_m \sin^2 2\theta}{\omega_m^2 - d^2} - \frac{2}{\omega_m} \right] G^2, \\
u_1 &= \frac{G^2 \sin^4 \theta}{\omega_f + d - \omega_m} + \frac{G^2 \cos^4 \theta}{\omega_f - d - \omega_m}, \\
u_2 &= -\frac{G^2 \sin^4 \theta}{\omega_f + d + \omega_m} - \frac{G^2 \cos^4 \theta}{\omega_f - d + \omega_m}, \\
s &= -\left[\frac{1}{\omega_m} + \frac{\omega_m \sin^2 2\theta}{2\omega_m^2 - 2d^2} \right] G^2, \\
\omega_f &= \omega_{c2} - \omega_{c1}.
\end{aligned} \tag{12}$$

The first term of the effective Hamiltonian (11) describes the cross-Kerr nonlinearity between the two cavity modes, and the second term is that between one of the cavity modes and the oscillator. The third term is the Kerr nonlinearity of cavity fields.

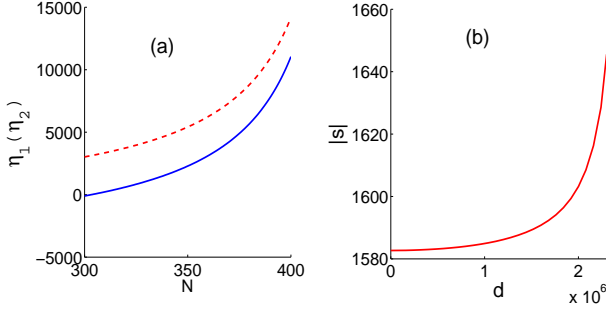


FIG. 2: (Color online) The enhancement of the strength of the cross-Kerr and the Kerr nonlinearity. (a): $\eta_1(\eta_2)$ is proportional to the number of atoms where $d = 200\pi KHz$. (b): $|s|$ change with d for $N = 350$. The other parameters are $g_1 = 20\pi KHz$, $g_2 = 22\pi KHz$, $\omega_m = 800\pi KHz$, $G = 20\pi KHz$, $\Omega = 20\pi KHz$, $\delta = \Delta = 250\pi KHz$.

In Fig.2a, we show that the cross-Kerr nonlinearity strength η_1 and η_2 increase with the increasing of the number of atoms at certain range of parameters, which means that the cross-nonlinearity can be enhanced by increasing of the number of atoms. From the coefficients of expression (12), we know that the reason of η_i changing with the number of atoms lies in the frequency difference ω_f between quasi-mode ω_{c1} and ω_{c2} modulated by the number of atoms N . Usually, it is difficult to adjust the

radiation pressure coupling G , because it is determined by the mirror and the cavity. Therefore, to enhance the cross-Kerr nonlinearity, we should decrease $|\omega_f \pm d \pm \omega_m|$. If $|\omega_f \pm d \pm \omega_m| < \omega_m$, the nonlinearity is larger than $\frac{G^2}{\omega_m^2}$ [22]. The frequency difference d between two modes and the frequency difference ω_f between quasi-mode ω_{c1} and ω_{c2} can favor us do that. We can achieve our target by controlling the number of atoms and keep an appropriate value of d . Of course, in the above process, we should keep $\{G \sin^2 \theta, G \cos^2 \theta\} < |\omega_f \pm d \pm \omega_m|$ so as to meet the condition of effective Hamiltonian approximation. Comparing with [23] where the tunneling rate between two cavities is the key factor to enhance the nonlinearity, the modulation of the nonlinearity by adjusting the number of atoms is easier to implement and control. Fig.2b shows us the strength of Kerr nonlinearity $|s|$ (in unit of Hz) as a function of d . Although s is not influenced by N (θ has nothing to do with N see Eq.(10)), the nonlinearity $|s|$ can be enhanced, because one of the denominators of s is decreased by d while this behavior can not exist just in single cavity optomechanical system [22]. Furthermore, because the photon-blocked effect of the third term, it is easy to perform QND of single photon, while this property has no exhibition in [23].

III. THE QUANTUM-NONDEMOLITION MEASUREMENT OF PHOTON

Cross-Kerr nonlinearity is believed as high efficiency quantum-nondemolition measurement [29, 30]. It also can be used to perform quantum gate [31, 32], quantum information procession [33, 34], and entanglement generation [35–38]. As a usage of the present scheme, we now study the QND measurement of phonon and photon.

The master equation of the system is

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}_I, \rho] + \sum_{i=1}^4 D[\hat{A}_i]\hat{\rho}, \tag{13}$$

where $\hat{H}_I = \hat{H}_{dr1} + \hat{H}_{af2} + \hat{H}_{fo1}$, $D[\hat{A}_i] = \hat{A}_i \rho \hat{A}_i^\dagger -$

$\frac{1}{2}\rho\hat{A}_i^\dagger\hat{A}_i - \frac{1}{2}\hat{A}_i^\dagger\hat{A}_i\rho$, $\hat{A}_1 = \sqrt{2\kappa_1}a_1$, $\hat{A}_2 = \sqrt{2\kappa_2}a_2$, $\hat{A}_3 = \sqrt{2\gamma_m(n_{th}+1)}\hat{b}$, and $\hat{A}_4 = \sqrt{2\gamma_m n_{th}}\hat{b}^\dagger$. In order to illustrate the nonlinearity, we employ the Hamiltonian in the interaction picture rather than using effective Hamiltonian (11). Due to the high frequency of the cavity and the large frequency difference between the cavity fields and the movable mirror, the environment of the cavity fields can be treated as zero temperature while the mirror should be in thermal field.

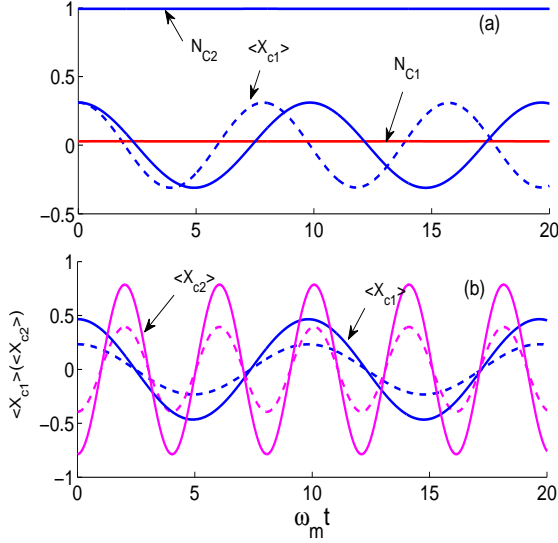


FIG. 3: (Color online) The existence of cross-Kerr nonlinearity via quadrature $\langle X \rangle$ measurement. (a): The evolution of $\langle X_{C1} \rangle$ for initial state $|0.1_{c1}, 1_{c2}\rangle$, with $N = 320$ (solid line) and 400 (dashed line). (b): $\langle X_{C1} \rangle$ and $\langle X_{C2} \rangle$ for two initial coherent states $|0.1_{c1}, 0.2_{c2}\rangle$ (dashed) and $|0.2_{c1}, 0.4_{c2}\rangle$ (solid line) where $N = 380$. The other parameters are the same with Fig.2.

Now we first illustrate the existence of cross-Kerr nonlinearity within the system. For simplicity, we omit the pumping fields and the loss of the fields and assume that the oscillator is in its ground state. In Fig.3a, c_1 is in coherent state $|\alpha\rangle$ ($\alpha = 0.1$), and the mode c_2 is initially in Fock state $|1\rangle$. If the system contains the cross-Kerr nonlinearity with the term $\eta_1 \hat{c}_1^\dagger \hat{c}_1 \hat{c}_2^\dagger \hat{c}_2$, then we will have $e^{-i\eta_1 \hat{c}_1^\dagger \hat{c}_1 \hat{c}_2^\dagger \hat{c}_2 t} |\alpha, 1\rangle_{c_1, c_2} = |\alpha e^{i\eta_1 t}, 1\rangle$, the c_1 mode acquires a phase. When the phase equals to π , a two-photon controlled-phase gate is naturally im-

plemented, from which a CNOT gate can also be easily constructed [31, 32]. Performing quadrature operator $x = c_1 + c_1^\dagger$ measurement by the homodyne apparatus, we know $\langle x \rangle = 2\alpha \cos \eta_1 t$. Employing the Hamiltonian $\hat{H} = \hat{H}_{aff1} + \hat{H}_{fo2}$, we plot $\langle \hat{c}_1^\dagger \hat{c}_1 \rangle$, $\langle \hat{c}_2^\dagger \hat{c}_2 \rangle$, $\langle X_{c1} \rangle$ and $\langle X_{c2} \rangle$ in Fig.3. We see that the photon number almost keeps unchanged but $\langle x \rangle$ oscillates with cosine function. In addition, the frequency of cosine function increases with the increasing of N (see Fig.3a), that is to say, the strength of the cross-Kerr nonlinearity enhances with the increasing of N . For both of the quasi-mode initially in coherent state, Fig.3b shows that the periods of the oscillation of $\langle X_{c1} \rangle$ and $\langle X_{c2} \rangle$ have nothing to do with different initial coherent state, only the amplitudes are affected by the coherent states. Although we plot the figure from $\hat{H} = \hat{H}_{aff1} + \hat{H}_{fo2}$, one can clearly see that cross-Kerr nonlinearity do exist in the two-mode optomechanical system.

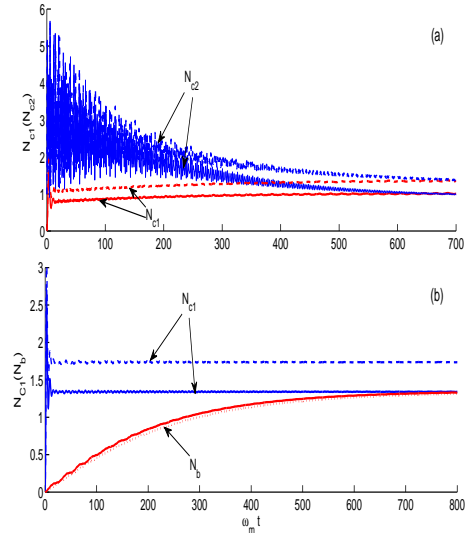


FIG. 4: (Color online) The QND of photon number as well as phonon number. The QND of N_{C2} via detection of N_{C1} (a) where $\varepsilon_1 = 23g_1$, $\varepsilon_2 = 62g_1$, and the QND of phonon number N_b via detection of N_{C1} (b) where $\varepsilon_1 = 31g_1$, $\varepsilon_2 = \varepsilon_1 \tan \theta$, $n_{th} = 4$. For both of the figures $\kappa_1 = 200\pi KHz$, $\kappa_2 = 0.01\kappa_1$, $\gamma_m = 0.001\kappa_1$, $N = 320$ (dashed line), 380 (solid line), and the other parameters are the same with Fig.2.

We now discuss the QND of photon and phonon num-

ber via detecting the photon number of c_1 . In Fig.4a, initially there is a single photon in mode c_2 , c_1 is in vacuum state, and the phonon is in thermal state with $T = 42\mu K$. In order to perform QND in mode c_2 , we need a weak classical driving field to compensate the loss of the cavity so as to keep the single photon, and we also need a little bit strong driving field to pump mode c_1 . Because of the detection of mode c_1 , we have assumed that the loss of mode c_1 is larger than that of c_2 and phonon mode. For the case $\cos\theta \approx 1$ in the present parameter region, the above condition by assuming $\kappa_1 \gg \kappa_2$ can be realized. Fig.4a clearly shows us that after a period of time evolution, the photon number of mode c_1 and c_2 are equal, which means that the leaking-out photon number of mode c_1 is exactly equal to that of mode c_2 ; thus we can perform QND of c_2 by detecting c_1 . If we want to detect phonon number, we need the phonon excitations which can be obtained by employing the thermal field. Fig.4b displays that under appropriate condition, the QND of phonon number also can be performed by detecting the same mode c_1 , where we assume that the mode c_1 and c_2 are both initially in vacuum state, the phonon mode is in thermal state with $T = 42\mu K$, and the mode c_2 is not driven under the condition $\varepsilon_2 = \varepsilon_1 \tan\theta$. From Fig.4, we conclude that two initial independent bosonic modes finally acquire identical intensity after evolution, which is the so called synchronization. In addition, the number of atoms does affect the behavior of synchronization. For less number of atoms, the time of evolution to achieve synchronization is longer than that with more atoms. Comparing dashed line ($N = 320$) with solid line ($N = 380$), we can clearly observe it. Thus, the number of atoms do modulate the nonlinearity. We can enhance the nonlinear strength by increasing the number of atoms in the present region of parameters. As we know that nonlinear interaction is necessary for the emergence of synchronization [39, 40]; therefore, we can safely say

that the cross-nonlinearity within the present scheme really can offer us QND photon and phonon numbers.

During the review process of the paper, we read the related works [41, 42]. For strongly driven optomechanical system, they included the nonlinear interaction term usually omitted by most works and shown that intrinsic nonlinear are observable even with a relatively weak optomechanical coupling. Different from [41, 42], we discuss the weakly driven and weak coupling condition, we do not employ the so called linearized optomechanics. Most importantly, we put forward an alternatively scheme to enhance the cross-Kerr nonlinearity via atomic coherence.

IV. CONCLUSION

We put forward a scheme employing atomic coherence to enhance the nonlinearity of optomechanical system. When the atoms interact with the two mode fields with large detuning condition, we adiabatically eliminated the degree of atoms. For weak coupling among the two mode fields and the mechanical resonator, we derived the effective Hamiltonian and shown that the nonlinear coefficients can be enhanced by increasing the number of atoms at certain range of parameters. We also numerically studied the nonlinearity enhancement beyond the effective Hamiltonian. Furthermore, we investigated the potential quantum nondemolition measurement of bosonic mode. Our results show that the present system exhibits synchronization, and the nonlinear effects provide us a means in performing QND.

As to the realizability in experiment, we do not demand the strong coupling of atoms with the cavity, for example, our parameters satisfy $g_1 \ll \kappa_1$, $g_2 \approx \kappa_2$. We also do not require strong coupling between the cavity fields and the mechanical resonator, and one can easily check that the so called strong coupling condition $G^2 > \kappa\omega_m$ is not meet. Although the above strong coupling conditions are achievable, the loosening of the coupling conditions

is more realizable and is still valuable.

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